

<b>Academic Year</b>	AY17/18	<b>Semester</b>	2
<b>Course Coordinator</b>	Dr Tan Hai Siong		
<b>Course Code</b>	PH4509		
<b>Course Title</b>	Quantum Field Theory with applications in Condensed Matter Physics		
<b>Pre-requisites</b>	PH3101 – Quantum Mechanics II		
<b>No of AUs</b>	4		
<b>Contact Hours</b>	52		
<b>Proposal Date</b>	27 September 2017		

**Course Aims**

The course is intended to be a first course in QFT, in which you should, upon completion, gain mastery of essential ideas in field theory and broadly appreciate how they relate to the deep aspects of fundamental physics. The course will enable you to develop an awareness of the plethora of applications ranging from particle theory to many-body problems in condensed matter physics. In the process, you will appreciate the universality of QFT and how it provides a fine example of the interdisciplinary nature of physics. The concrete examples invoked in particle physics and condensed matter applications will enable you to learn how abstract principles in the study of physics are intimately related to concrete physical observables. It will be a course where you learn to navigate through mathematical methods in your own stride without losing sight of the underlying physical narrative. It should very much widen your technical repertoire and deepen the maturity of your physical intuitions.

**Intended Learning Outcomes (ILO)**

By the end of this course, you (as a student) would be able to:

<p>[A] Aspects of relativistic quantum theory</p>	<p><i>To be able to</i></p> <ol style="list-style-type: none"> <li>1. Explain basic aspects of the free Klein-Gordan equation and the Dirac equation in the following areas           <ul style="list-style-type: none"> <li>• Construct and interpret plane-wave solutions</li> <li>• Describe the Dirac matrices as representations of the Clifford algebra and use them to construct generators of Lorentz transformations</li> <li>• Recognize and demonstrate, using for example the covariant property of Dirac spinors, the Lorentz invariance of these equations</li> <li>• Describe and explain the problems in interpreting them as consistent relativistic wave equations</li> </ul> </li> <li>2. Recognize various elements of the Lorentz algebra in terms of generators of rotations and boosts and to demonstrate how it can be interpreted as a product of two <math>su(2)</math> algebras.</li> <li>3. Distinguish between the Heisenberg, Dirac and Schrodinger pictures.</li> <li>4. Relate between generators of infinitesimal symmetry transformations and the operators representing the corresponding finite ones.</li> </ol>
<p>[B] Canonical quantization and Feynman diagrams</p>	<p><i>To be able to</i></p> <ol style="list-style-type: none"> <li>1. Draw Feynman diagrams up to some order in the number of vertices that corresponds to both correlation functions and S-matrix elements.</li> </ol>

	<p>2. Define the notions and give illustrative examples of connected, fully connected, amputated diagrams for S-matrix elements and vacuum bubbles.</p> <p>3. Explain what the Feynman diagrams correspond to, and relate them to the time-ordered product of quantum fields via Wick's theorem.</p> <p>4. Demonstrate how the Feynman rules emerge from the Lagrangian and use them to evaluate the diagrams.</p> <p>5. State and prove Wick's theorem, and explain the significance of the theorem in perturbative quantum field theory.</p> <p>6. Derive propagators which are Green's functions of the Klein-Gordon operator subject to different boundary conditions, and to demonstrate how they could be represented on the complex plane via a deformed contour.</p> <p>7. Define the concept of a field operator, eg. how it can be expressed in terms of creation and annihilation operators, and canonical quantization.</p>
<p>[C] Path-Integrals in QFT</p>	<p><i>To be able to</i></p> <p>1. Demonstrate how correlation functions of scalar fields can be obtained by taking functional derivatives of a generating functional, and compute them explicitly for a free scalar field theory. Explain how one can generalize such a computation to an interacting field theory.</p> <p>2. Define the notions of functional derivatives and integration, and state how they are important as part of the mathematical language of QFT.</p> <p>3. Recognize the global symmetry corresponding to a phase transformation of the scalar field in complex scalar field theory and compute the conserved charge and current as a simple example of Noether's theorem in QFT.</p>
<p>[D] On Quantum Hall Effect</p>	<p><i>To be able to</i></p> <p>1. Derive degeneracy of Landau levels using magnetic translation operators and demonstrate quantization of magnetic flux for a simple model of quantum hall fluid on a torus.</p> <p>2. Explain the idea of how toroidal boundary conditions are used to determine the single particle and multi-particle wavefunctions, for example how one can derive the quantized Hall conductance from Niu-Thouless-Wu formula</p> <p>3. Describe qualitatively the effective field theory approach towards deriving the Fractional Quantum Hall Effect.</p> <p>4. Use the Zhang-Hansson-Kivelson action as the starting point, derive the fractional quantum hall effect by integrating out certain degrees of freedom.</p>

**Course Content**

This is a first course in quantum field theory (QFT) where we provide an exposition of basic concepts, mathematical techniques and a sketch of various applications in particle theory and many-body

problems in condensed matter physics. The foundational topics which will be introduced include path-integral formalism in quantum mechanics and QFT, canonical quantization, Green's functions and Feynman diagrams in perturbation theory. We will touch on the application of these concepts to aspects of quantum electrodynamics as well as selected modern topics in condensed matter physics for which QFT is a useful framework, such as the fractional quantum hall effect, mean-field theory of superfluids, renormalization group and Landau-Ginzburg theory of critical phenomena.

**Assessment (includes both continuous and summative assessment)**

Component	Course LO Tested	Related Programme LO or Graduate Attributes	Weighting	Team/Individual	Assessment Rubrics
1. Homework/ Assignments	All stated	1. Competency 2. Creativity	25%	Individual	Appendix 1
2. Midterm Tests	MT1: all course objectives listed in [A] and [B]. MT2: all course objectives listed in [C] and [D]	1. Competency 2. Creativity	50%	Individual	Appendix 2
3. Project	All stated	1. Competency 2. Creativity 3. Communication skills	25%	Individual	Appendix 3
			100%		

**Formative feedback**

The homework assignments will allow me to understand how each student is progressing along, and for me to correct any misconceptions and identify specific topics which are not well-understood. Feedback will be reflected on their homework scripts and if necessary, extra consultation hours will be arranged to address learning difficulties. During tutorial sessions, students will be invited to present solutions and feedback will be given during their participation. I will also react to their project presentation in terms of encouraging remarks and a critical evaluation of content presented.

**Learning and Teaching approach**

Approach	How does this approach support students in achieving the learning outcomes?
Lecture	During lectures, the teaching approach would be mainly in the form of whiteboard presentation suitable for the nature of this topic (it is mathematically rather intensive and the traditional board work fits best), as well as seminar-style discussion of certain video materials such as interviews of physicists, etc. This would encourage students to not merely pick up the hard technical aspects of the course content, but also to develop a more holistic awareness of the social fabric of the community of physicists who are working actively in this field of physics, and various

	aspects of the art and philosophy of doing science.
Tutorials	During tutorials, students will learn and understand various approaches of solving problems which would assist them to internalize the concepts presented in lectures. Their participation in presenting their own solutions will also refine their communication skills at the same time clarifying their own understanding of the course materials.

### Reading and References

- a. \*Zee A., 2010, *Quantum Field Theory in a Nutshell*, Princeton University Press. ISBN: 9781400835324
- b. Srednicki M., 2007, *Quantum Field Theory*, Cambridge University Press. ISBN: 0521864496
- c. \*Fradkin E., 2013, *Field Theories of Condensed Matter Physics*, Cambridge University Press. ISBN: 9780521764445
- d. Wen X.G., 2007, *Quantum Field Theory of Many-Body Systems*, Oxford University Press. ISBN: 0198530943
- e. Greiner W. and Reinhardt J., 1996, *Field Quantization*, Springer Science. ISBN: 9783642614859
- f. Coleman S., 1985, *Aspects of Symmetry*, Cambridge University Press. ISBN: 978-0521318273
- g. \*Peskin M.E. and Schroeder D., 1995, *Introduction to Quantum Field Theory*, Westview Press. ISBN: 978-0201503975
- h. Cardy J., 1996, *Scaling and renormalization in statistical physics*, Cambridge Lecture Notes in Physics. ISBN: 978-0521499590.
- i. Banks T., 2008, *Modern Quantum Field Theory - A concise introduction*, Cambridge University Press. ISBN: 978-0521850827

\*References **a,c** and **g** will be used as the course textbooks.

### Course Policies and Student Responsibilities

#### (1) General

Students are expected to complete all homework assignments, submit them punctually and attend all classes. They are expected to be responsible for their own learning, to adopt an active approach towards understanding the course materials and to view assignments, midterms and project seriously. They are expected to behave well in class and be respectful to both their peers and the course instructor.

#### (2) Compulsory homework assignments

Students are required to hand in mandatory homework assignments on or before each specified due date. Plagiarism will be taken very seriously.

### Academic Integrity

Good academic work depends on honesty and ethical behaviour. The quality of your work as a student relies on adhering to the principles of academic integrity and to the NTU Honour Code, a set of values

shared by the whole university community. Truth, Trust and Justice are at the core of NTU's shared values.

As a student, it is important that you recognize your responsibilities in understanding and applying the principles of academic integrity in all the work you do at NTU. Not knowing what is involved in maintaining academic integrity does not excuse academic dishonesty. You need to actively equip yourself with strategies to avoid all forms of academic dishonesty, including plagiarism, academic fraud, collusion and cheating. If you are uncertain of the definitions of any of these terms, you should go to the [academic integrity website](#) for more information. Consult your instructor(s) if you need any clarification about the requirements of academic integrity in the course.

### Course Instructors

Instructor	Office Location	Phone	Email
Dr Tan Hai Siong	PAP 02-01	86506378	hs.tan@ntu.edu.sg

### Planned Weekly Schedule

Week	Topic	Course LO	Readings/ Activities
	[Lecture and tutorials]	As described above	[Please refer to labels defined in the reading list above]
Weeks 1-2	Review of special relativity; path integrals in quantum mechanics; symmetries in quantum theory; review of variational calculus	[A1], [A2],[A3],[A4]	References (a), (b), (e), (g).
Weeks 3-4	Path-integrals in quantum field theory	C[1],C[2],C[3]	References (a),(b),(e),(g),(i)
Weeks 5-6	Feynman diagrams and Green's functions in relativistic quantum field theory	C[1],C[2],C[3]	References (a),(b),(g)
Weeks 7-9	Canonical quantization and the Fock space; understanding the QFT vacuum; many-body physics and some simple examples in condensed matter theory	B[1],B[2],B[3], B[4],B[5],B[6], B[7]	References (a),(b), (c), (d), (e), (g)

Weeks 9-10	Application 1: Quantum electrodynamics with a broad brush; gauge field theories in particle physics	C[1],C[2],C[3]	References (a),(b),(g)
Weeks 11-13	Application 2: Field theories in condensed matter physics <ul style="list-style-type: none"> <li>• Linear response theory</li> <li>• Quantum Hall Fluids, FQHE and Chern-Simons field theory</li> <li>• Renormalization Group</li> <li>• Mean-field theory of superfluid</li> <li>• Basic aspects of conformal field theory</li> <li>• Landau-Ginzburg theory of critical phenomena</li> </ul>	D[1],D[2],D[3], D[4]	References (a),(c),(d),(f),(g),(h),(i)

Appendix 1: Assessment Criteria for Homework/ Assignments

Each homework assignment consists of a variety of questions relevant to the lecture content. They will be mostly technical questions designed to assist students assimilate various concepts presented in lectures. There will also be a few short reading assignments designed to invite students to critically respond to semi-technical articles written by or about quantum field theorists. The mark percentages for each question will be indicated and the points given to each question will be based not only on the precision of the final answer but also on the validity of the approach.

By mark range

Marks	Criteria
> 90%	Demonstrates very complete understanding of the various conceptual and technical aspects of quantum field theory; be able to solve relevant questions reflecting understanding of the topic at the level of being able to apply to advanced problems in particle theory and condensed matter theory (as described in the intended learning outcomes)
75% to 89%	Demonstrates ability to solve a variety of questions in quantum field theory and a sound awareness of the central principles of QFT. In other words, you have demonstrated the achievement of the learning outcomes in a comprehensive way.
65% to 74%	Able to solve most elementary questions with partially consistent and valid attempts in applying course content to unfamiliar questions. In other words, you have demonstrated the potential to achieve the learning outcomes in time.

50% to 64%	Able to solve only the most basic questions relevant to the course content. Has difficulty in applying ideas to new contexts. Lack of clarity in physical interpretations. In other words, you have demonstrated an achievement of some of the learning outcomes and have some way to go with the others.
< 50%	Unable to solve most basic questions relevant to the course content, and lack of basic mathematical techniques to apply concepts presented in lectures and described in the learning outcomes to new contexts and problems.

#### Appendix 2: Assessment Criteria for Midterm Tests

Two midterm tests each planned to be 90 minutes, consisting of questions similar in rigor and style to tutorial questions, and the range of questions to appropriately reflect the content coverage of the lectures.

#### By mark range

Marks	Criteria
> 90%	A complete and impressive demonstration of the achievement of the learning outcomes.  Successful and impressive attempts at solving all the questions, including bonus questions. Demonstrates mastery of the various conceptual and technical aspects of quantum field theory; be able to solve relevant questions reflecting understanding of the topic at the level of being able to apply to advanced problems in particle theory and condensed matter theory.
75% to 89%	A strong demonstration of the achievement of the learning outcomes  Correct solutions to most of the questions in the assignments. Demonstrates ability to solve a variety of questions in quantum field theory and a sound awareness of the central principles of QFT.
65% to 74%	A decent demonstration of the achievement of the learning outcomes.  Able to solve most elementary questions with partially consistent and valid attempts in applying course content to unfamiliar questions.
50% to 64%	A superficial demonstration of the achievement of the learning outcomes  Able to solve only the most basic questions relevant to the course content. Has difficulty in applying ideas to new contexts. Lack of clarity in physical interpretations.
< 50%	A insufficient demonstration of the achievement of the learning outcomes  Unable to solve most basic questions relevant to the course content, and lack of basic mathematical techniques to apply concepts presented in lectures to new contexts and problems.

#### Appendix 3: Assessment Criteria for Project

The project component's assessment consists of two parts: the report and the presentation. The report will be graded based on the depth and breadth of analysis manifest in the literature review and bonus points will be awarded to the demonstration of original work based on the project theme. In the oral presentation component, the grade will reflect the ability of students to communicate

and explain their ideas across to the audience, their response to spontaneous questions and the clarity of their overall presentation.

[By mark range](#)

Marks	Criteria
> 90%	<p>A complete and impressive demonstration of the achievement of the learning outcomes.</p> <p>Project report indicates excellent understanding of the scientific article(s); demonstrates not only an overall solid understanding of the broad ideas in the literature review but also some of the important concrete details to the extent of being able to reproduce equations and conclusions in the scientific articles; demonstrate ability to critically evaluate articles; demonstrates some form of original work built upon the known results in the literature review.</p> <p>Oral presentation demonstrates clear and strong ability to communicate various concepts in the literature review with high amount of clarity in presentation, a sound understanding of physical ideas and conclusions of various scientific articles. Student is able to comment constructively on the importance of various results. Presentation style is interesting and fluent. Able to respond coherently to questions.</p>
70% to 89%	<p>A strong demonstration of the achievement of the learning outcomes</p> <p>Project report demonstrates that the student read and understood most of the concepts presented in the scientific articles. There is some fair attempt in critically responding to the conclusions of the articles and not just a mere summary of facts.</p> <p>Oral presentation shows excellent ability in communicating various concepts at various levels, and genuine understanding of the scientific articles. Able to respond fairly and soundly to most questions.</p>
60% to 69%	<p>A valiant effort to achieve the learning outcomes</p> <p>Project report demonstrates a devoted attempt in covering the ideas presented in the scientific articles, but review lacks in depth and clarity. There is some modest and reasonable attempt in responding to conclusions of various articles more than a cursory summary of the facts presented in the papers, demonstrating some form of original thoughts.</p> <p>Oral presentation shows effort in presenting a summary of results in the various scientific articles; unable to respond to some (basic) questions showing possible lack of rigorous grasp of key ideas in the literature review.</p>
50% to 59%	<p>A weak attempt at achieving the learning outcomes</p> <p>Project report shows a minimal summary of the results of various papers, put together without a strong sense of organization. Very little original response to the content of the articles in the form of critical evaluation.</p>

	<p>Oral presentation lacks strong structural coherence and inability to convey clearly the results of various articles. But there is a decent attempt to furnish a summary containing basic ideas; unable to respond to most basic questions.</p>
< 50%	<p>An insufficient demonstration of the achievement of the learning outcomes</p> <p>Project report is very poorly written with a depth that is far less than what is expected if basic concepts covered in this course have been properly assimilated. Very little or complete lack of critical evaluations of the papers.</p> <p>Oral presentation is very weak, with signs of confusion indicating a weak grasp of the basic physical ideas in the literature review; unable to respond to basic questions.</p>